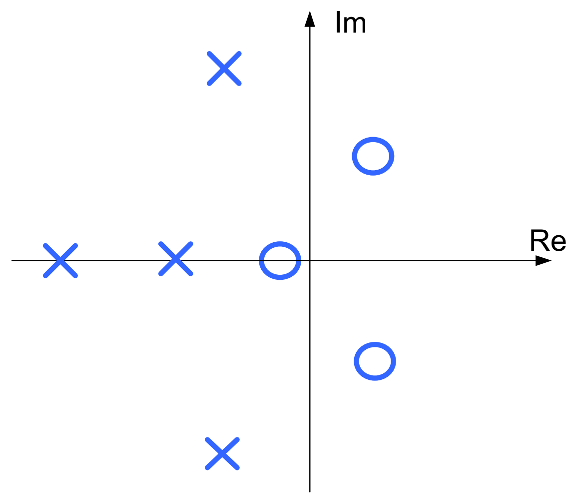
EE 230
Lecture 27

Waveform Generators

- Sinusoidal Oscillators

Review from Last Time:

Theorem: Any linear network that has a pole (or zero) at $\alpha_k+j\beta_k$ also has a pole (or zero) at $\alpha_k-j\beta_k$ whenever $\beta_k \neq 0$

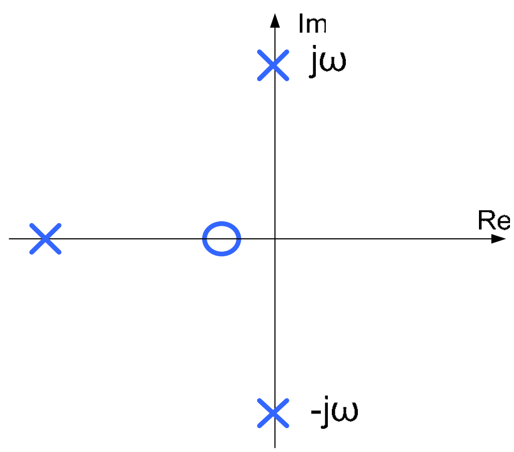


Poles and zeros always occur in complex conjugate pairs

Mathematically, the time domain response due to the combined effects of every complex conjugate pair of poles will always be real even though the time domain response due to a single complex pole may have an imaginary component.

Review from Last Time:

A Criteria for Sinusoidal Oscillation: A network that has a single complex conjugate pole pair on the imaginary axis at $\pm j\omega$ and no RHP poles will have a sinusoidal output of the form $X_O(t)=A\sin(\omega t+\theta)$

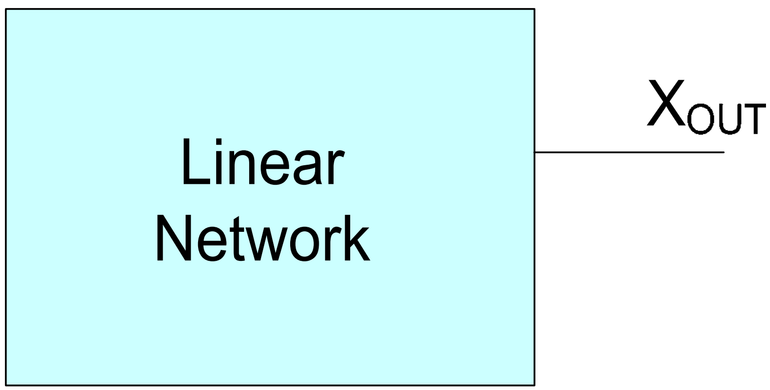


A and θ CAN NOT be determined from the properties of the linear network!

What properties of a circuit are needed to provide a sinusoidal output?

- Insight into how a sinusoidal oscillator works
- Characteristic Equation Requirements for Sinusoidal Oscillation (Sec 13.1)
- Barkhausen Criterion (Sec 13.1)

- Characteristic Equation Requirements for Sinusoidal Oscillation



Characteristic Equation Oscillation Criteria

If the characteristic equation D(s) has exactly one pair of roots on the imaginary axis and no roots in the RHP, the network will have a sinusoidal signal on every nongrounded node

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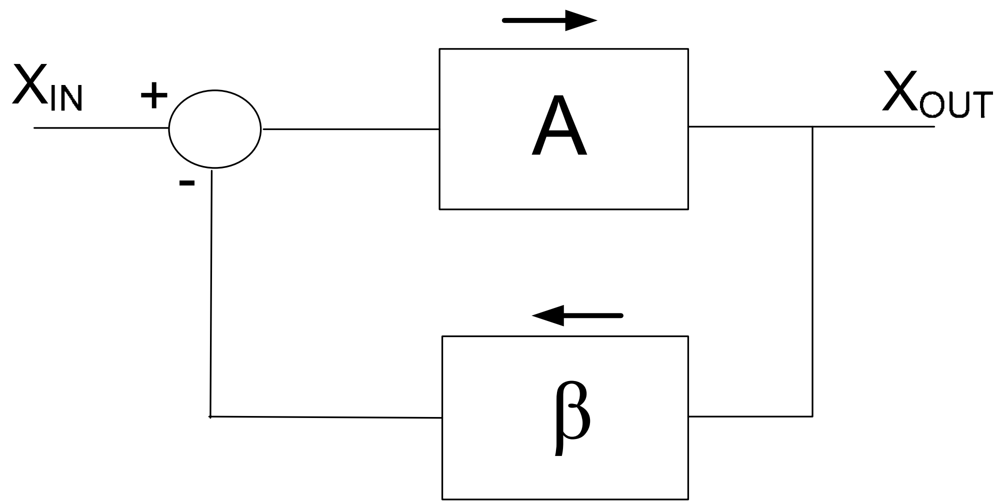
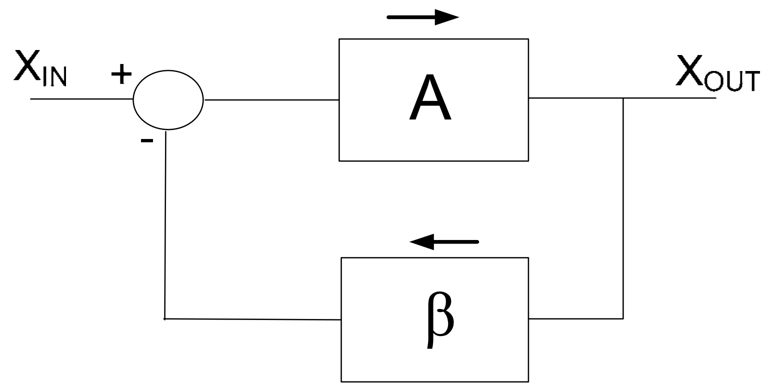
Barkhausen Criteria for Oscillation



$$A_{FB}(s) = \frac{A}{1 + A\beta}$$

$A\beta$ termed the loop gain

Barkhausen Criteria for Oscillation



$$A_{FB}(s) = \frac{A}{1 + A\beta}$$

Barkhausen Oscillation Criteria

A feedback amplifier will have sustained oscillation if $A\beta = -1$

Barkhausen Criteria often given in different ways by different authors. There are nontrivial subtle differences in how it is presented and not all are correct.

Barkhausen Criteria for Oscillation

Barkhausen Oscillation Criteria

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- Barkhausen Criteria often given in different ways by different authors.
- There are nontrivial subtle differences in how it is presented and not all are correct.
- Some make comments about shape of waveform (sinusoidal)
- Some add comments that must have $A\beta = -1$ at only one frequency
- Author is not rigorous (or correct) on definition of Barkhausen Criteria

Relationship between Barkhausen Criteria for Oscillation and Characteristic Equation Criteria

Barkhausen Oscillation Criteria

A feedback amplifier will have sustained oscillation if $A\beta=-1$

Characteristic Equation Oscillation Criteria

If the characteristic equation D(s) has exactly one pair of roots on the imaginary axis and no roots in the RHP, the network will have a sinusoidal signal on every nongrounded node

Differences:

1. Barkhausen requires a specific feedback amplifier architecture

2. Sustained oscillation says nothing about waveshape

Relationship between Barkhausen Criteria for Oscillation and Characteristic Equation Criteria

Barkhausen Oscillation Criteria

A feedback amplifier will have sustained oscillation if $A\beta=-1$

Characteristic Equation Oscillation Criteria (CEOC)

If the characteristic equation D(s) has exactly one pair of roots on the imaginary axis and no roots in the RHP, the network will have a sinusoidal signal on every nongrounded node

If a network can be represented as a basic feedback amplifier, then

$$A_{FB}(s)=\frac{A}{1+A\beta}=\frac{N(s)}{D(s)}$$

If D(s) criteria is satisfied, then at the poles $p=\pm j\omega$, $1+A\beta=0$ or equivalently $A\beta=-1$

But, if a network has $A\beta=-1$, even at a single pole pair, there may be other poles in the RHP that would violate the CEOC needed for sinusoidal oscillation

Relationship between Barkhausen Criteria for Oscillation and Characteristic Equation Criteria

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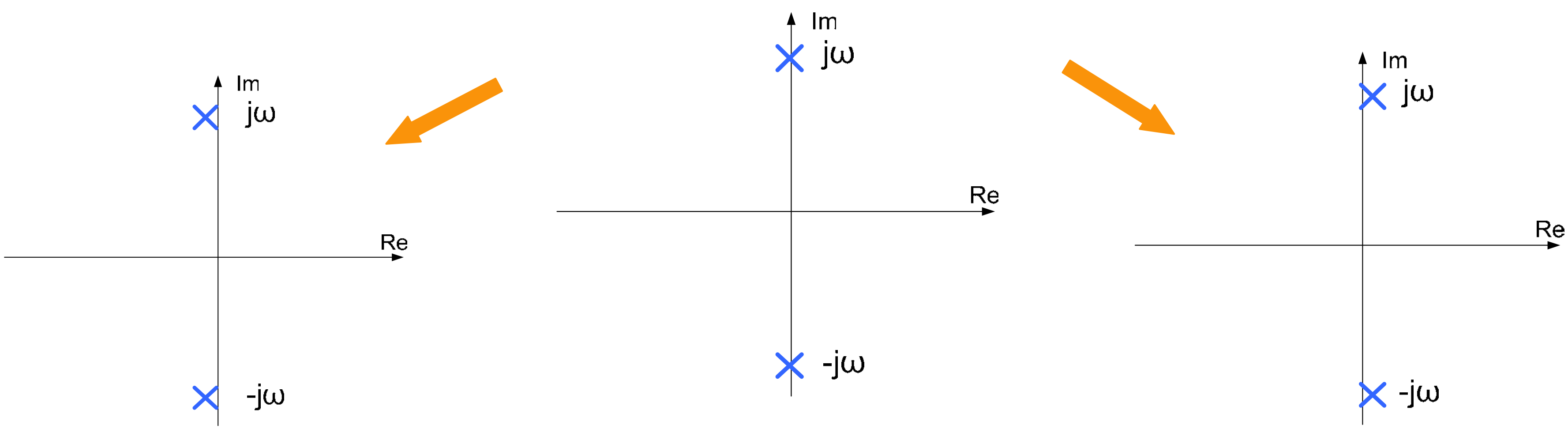
If sinusoidal oscillation is required, be careful when using Barkhausen Criteria

Characteristic Equation Oscillation Criteria (CEOC)

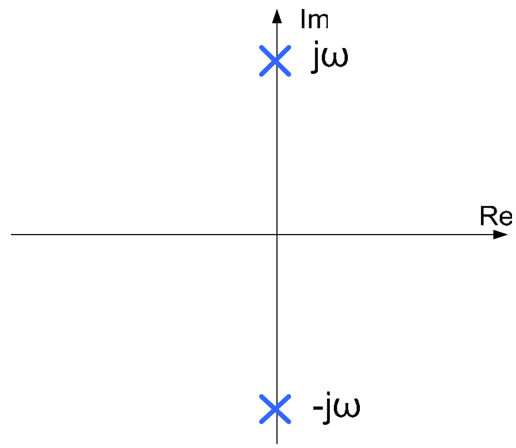
If the characteristic equation D(s) has exactly one pair of roots on the imaginary axis and no roots in the RHP, the network will have a sinusoidal signal on every nongrounded node

But – it is impossible to place a pair of poles of D(s) precisely on the imaginary axis !



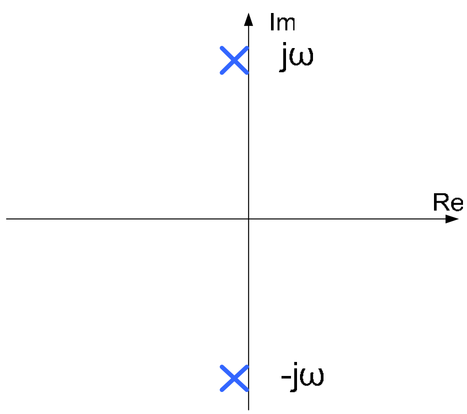
Characteristic Equation Oscillation Criteria (CEOC)

But – it is impossible to place a pair of poles of D(s) precisely on the imaginary axis ! If $p=\alpha \pm j\beta$



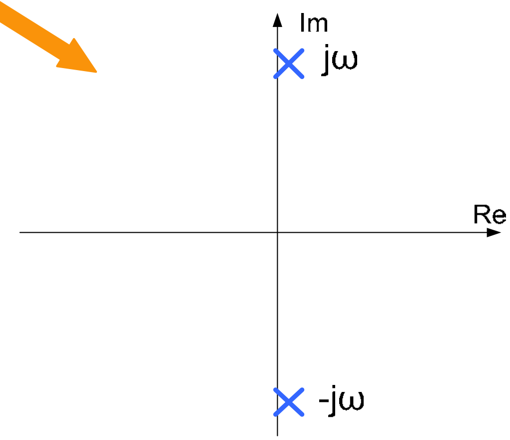
$$X(t)=A\sin(\omega t+\theta)$$

Impossible to achieve



$$X(t)=e^{\alpha t}A\sin(\omega t+\theta)$$

Sinusoidal signal will decay to 0



$$X(t)=e^{\alpha t}A\sin(\omega t+\theta)$$

Output will diverge to ∞

Characteristic Equation Oscillation Criteria (CEOC)

If the characteristic equation D(s) has exactly one pair of roots on the imaginary axis and no roots in the RHP, the network will have a sinusoidal signal on every nongrounded node

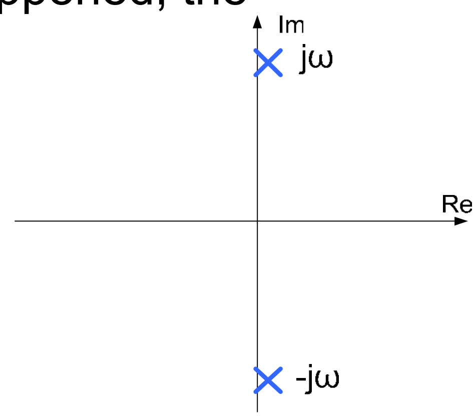
Sinusoidal Oscillator Design Strategy

Build networks with exactly one pair of complex conjugate roots slightly in the RHP and use nonlinearities in the amplifier part of the network to limit the amplitude of the output i.e. $p=\alpha \pm j\beta$ α very small but positive

Nonlinearity will result in a small amount of distortion

Frequency of oscillation will deviate slightly from β

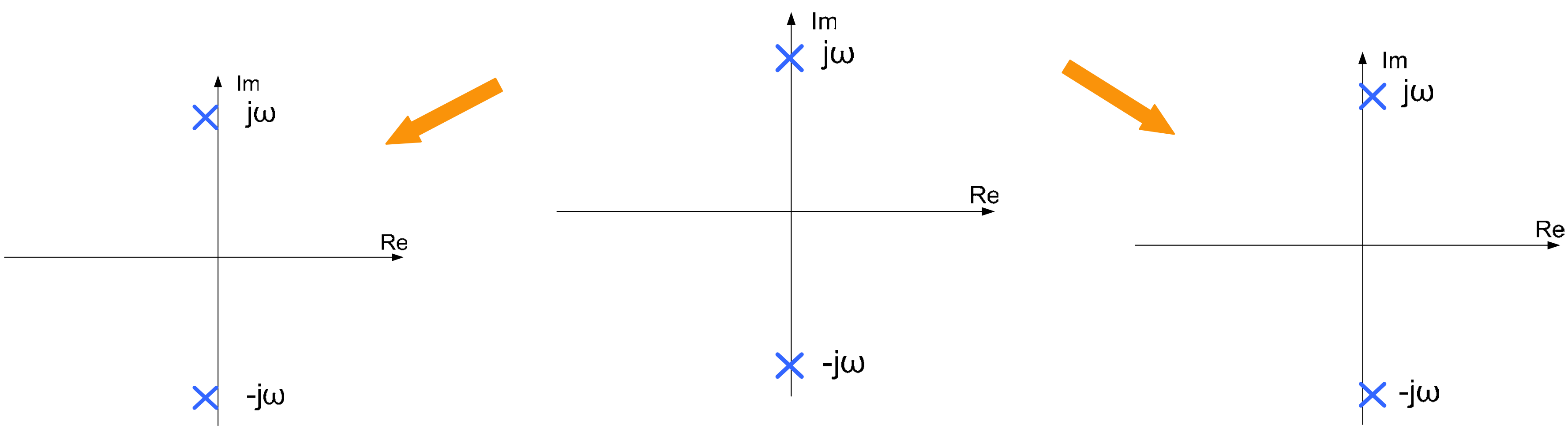
Must be far enough in the RHP so that process and temperature variations do not cause movement back into LHP because if that happened, the oscillation would cease !



Know Barkhausen Criteria to satisfy interviewers questions but use CEOC to design sinusoidal oscillators

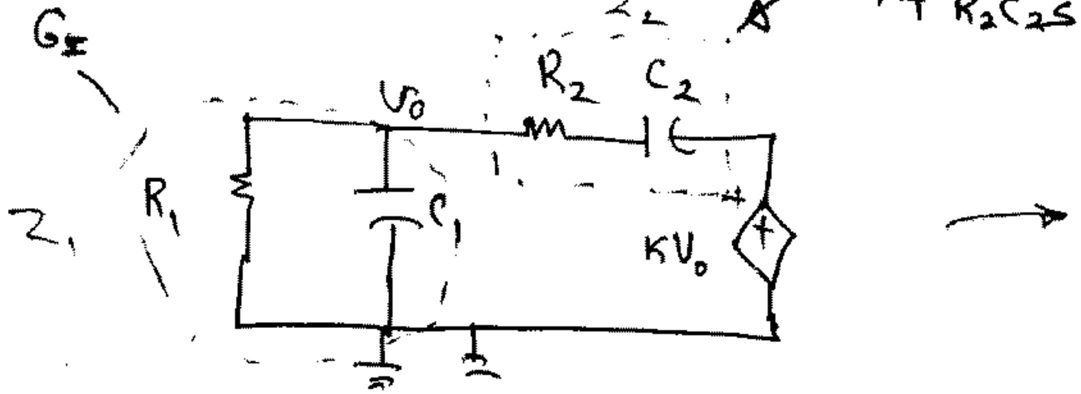
Relationship between Barkhausen Criteria for Oscillation and Characteristic Equation Criteria

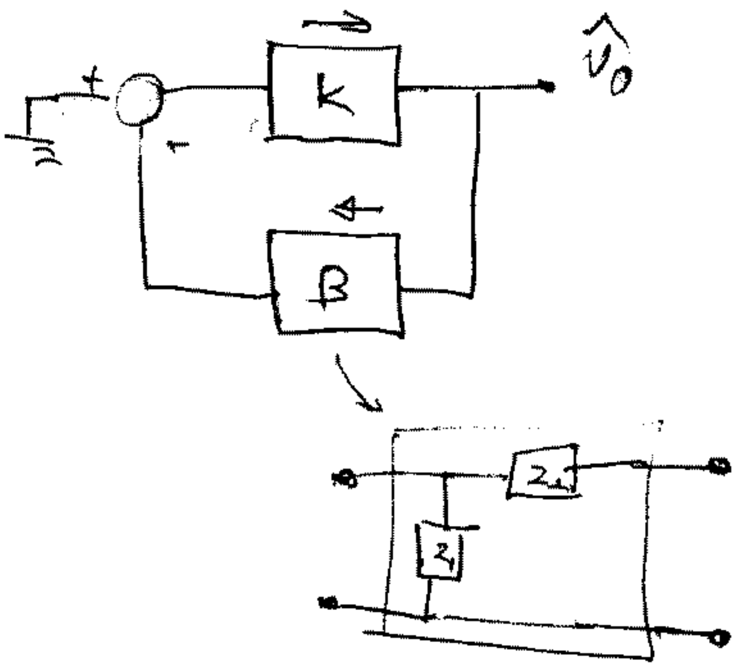
But – it is impossible to place a pair of poles of D(s) precisely on the imaginary axis !



Consider

$$\frac{sC_2}{1+R_2C_2s} = G_F$$





$$V_0\left(G_I + G_F\right) = K V_0 G_F$$

$$V_0\left(G_1 + sC_1 + \frac{sC_2}{1+R_2C_2s}\right) = K V_0 \frac{sC_2}{1+R_2C_2s}$$

$$V_0\left(s^2 + s\left[\frac{1}{R_2C_2} + \frac{1}{R_1C_1} + \frac{(1-K)}{R_2C_1}\right] + \frac{1}{R_1R_2C_1C_2}\right) = 0$$

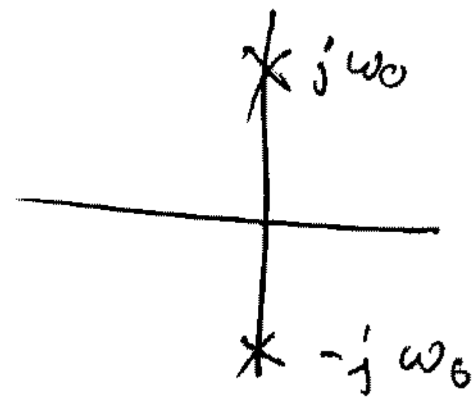
$$D(s) = s^2 + s\left[\frac{1}{R_2C_2} + \frac{1}{R_1C_1} + \frac{(1-K)}{R_2C_1}\right] + \frac{1}{R_1R_2C_1C_2}$$

To place the two poles on imaginary axis

set $$\frac{1}{R_2C_2} + \frac{1}{R_1C_1} + \frac{(1-k)}{R_2C_1} = 0$$

then

$$\omega_0 = \sqrt{\frac{1}{R_1R_2C_1C_2}}$$



Example : $R_1 = R_2 = R$, $C_1 = C_2 = C$

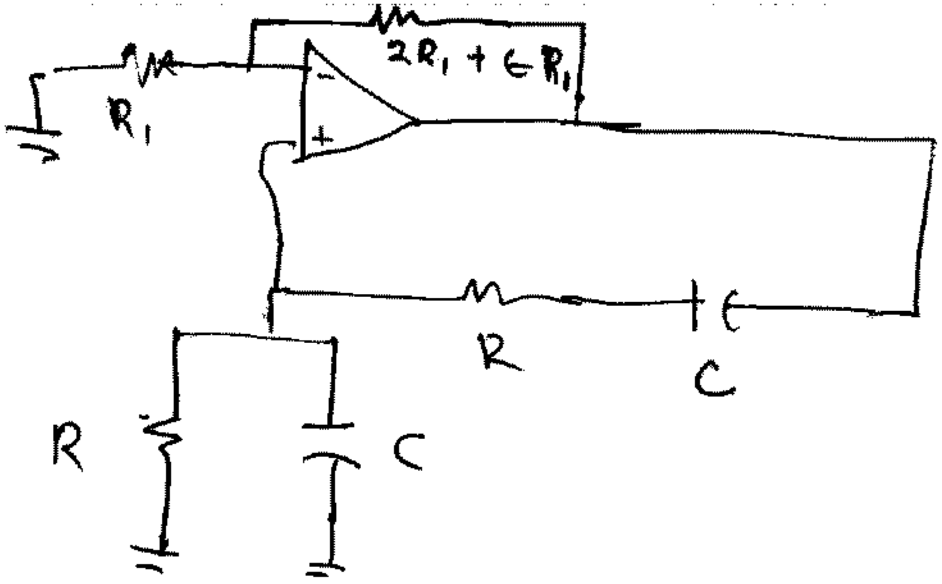
∠. Osc. criteria becomes

$$\frac{1}{RC}(3-k) = 0$$

or

$$K = 3$$

Practically, make $K = 3 + \epsilon$ to move poles slightly into RHP



Wein - Bridge Oscillator !

Observe the amplifier gain changes from greater than 3 to 0 at the discontinuity in derivative